TANGENTIAL STRESS IN THE DESCENDING FLOW OF A TURBULENT FILM AND A GAS

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UDC 532.5.536.24

The relationship between the pressure gradient and the tangential stress at the wall of the tube and at the interface is established. The tangential stress is found experimentally as a function of the Reynolds numbers of the two phases.

Many technological processes involve heat and mass transfer in turbulent films moving along with a gas. Hydrodynamic and thermal calculations for such flows require knowledge of either the velocity field or the hydraulic drag law. In either case it is necessary to know the tangential stress at the wall and at the interface. There has not been an adequate study of this topic for annular two-phase film flows. A complete analytic description of such flows is extremely difficult; in particular, it is not possible to analytically calculate the frictional stress at the interface, since the interaction between the liquid film and the gas leads to an irregular, wavy interface, which we do not understand well. There are also definite difficulties in directly measuring the tangential stress in a two-phase flow. A more convenient way to determine the tangential stress is to calculate it from the measured pressure gradient. This approach requires calculation of the relationship between the pressure gradient and the tangential stress at the wall and at the surface of the film.

Axisymmetric flow of a turbulent film and a gas in a vertical tube can be described by [1, 2]

$$-\frac{\partial P}{\partial x}(R-\varepsilon)+\frac{\partial}{\partial \varepsilon}[(R-\varepsilon)\tau_i]+[(2-i)\rho_i-(-1)^{1-i}\rho_2]g(R-\varepsilon)=0. \tag{1}$$

To save space Eq. (1) is written for both the liquid (i = 1) and the gas (i = 2).

The boundary conditions on (1) are

$$\tau_i=\tau_0 \text{ at } \varepsilon=0 \text{ and } \tau_i=\tau_\delta \text{ at } \varepsilon=\delta. \tag{2}$$

If we write (1) for the liquid component (i = 1) and integrate it over ε from 0 to δ, and then write Eq. (1) for the gas component (i = 2) and integrate it over ε from δ to R, then by combining the results we find a relationship between the pressure gradient and the tangential stress at the wall and at the interface:

$$\tau_0=-\frac{R}{2}\frac{\partial P}{\partial x}+(\rho_1-\rho_2)g\delta\frac{2R-\delta}{2R}+\rho_2 g\frac{(R-\delta)^2}{2R}, \tag{3}$$

$$\tau_\delta=\left(\rho_2 g-\frac{\partial P}{\partial x}\right)\frac{R-\delta}{2}. \tag{4}$$

To determine the tangential stress from Eqs. (3) and (4), we need experimental data on the pressure gradients and film thicknesses. Experiments have been reported [3] on the pressure drops for various regimes of an annular, descending air–water flow in a tube 30 mm in diameter and 2380 mm long. The pressure was sampled at five points along the length of the tube. In the same regions an electrical-contact

Vinnitsa Polytechnical Institute. Kiev Technological Institute of the Food Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 6, pp. 1050-1054, June, 1975. Original article submitted June 11, 1974.

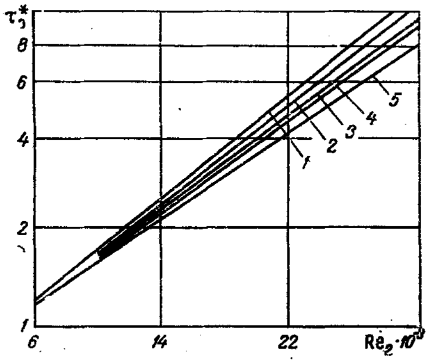

Fig. 1

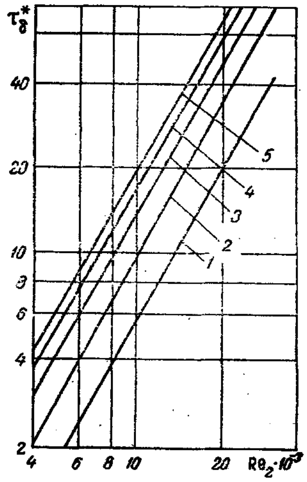

Fig. 2

Fig. 1. Dimensionless frictional stress at the wall as a function of the Reynolds number of the gas. 1) $Re_1 = 4000$; 2) 8000; 3) 12,000; 4) 16,000; 5) 20,000.

Fig. 2. Dimensionless tangential stress at the interface of various Reynolds numbers of the gas and liquid. The notation is the same as in Fig. 1.

method was used to measure the thickness of the liquid film for various ranges. In addition, the average thickness of the film in the tube was estimated for specified liquid and gas flow rates by using special high-speed shut-off valves. The average phase velocity was calculated from the measured flow rates.

On the basis of the pressure gradients determined experimentally, we calculated the tangential stress at the wall and at the interface. Figure 1 shows the tangential stress at the wall as a function of the Reynolds number of the gas. The dimensionless frictional stress in Fig. 1 was determined from

$$\tau_0^* = \tau_0/\rho_1 g\delta. \tag{5}$$

Here the tangential stress at the wall of the two-phase flow is divided by the stress corresponding to the liquid film of the same thickness draining under the influence of gravity. The experimental results show that for Reynolds numbers of the gas flow below 6000 the influence of this flow on the hydrodynamic characteristics of the turbulent film flow is extremely slight. For this reason, the values of the dimensionless tangential stress τ_0^* in Fig. 1 are approximately equal to one at $Re_2 \le 6000$. For $Re_2 > 6000$, the function $\tau_0^* = f(Re_2)$ becomes stronger. At $Re_2 > 14000$ this dependence is observed to be governed by the Reynolds number of the liquid; this effect becomes more appreciable as the Reynolds number of the gas is increased. At a fixed gas flow rate, relatively large numbers Re correspond to relatively small values of the dimensionless frictional stress. This circumstance is related to the large (in comparison with the pressure gradient) increment in the quantity $\rho_1 g\delta$ due to the increase in the film thickness as the Reynolds number of the liquid is increased. The dependence shown in Fig. 1 is quite complicated; it can be approximated by

$$\tau_0^* = \exp\left[(0{,}943 \cdot 10^{-4} - 3.56 \cdot 10^{-14} Re_1^2)\, Re_2 + (1.83 \cdot 10^{-10} Re_1^2 - 0.373)\right]. \tag{6}$$

In a two-phase annular flow the gas has a velocity gradient due to friction of the gas with the liquid film, and it has a flow velocity high in comparison with that of the liquid. The magnitude of the velocity gradient depends on the nature of the induced gas flow and the flow rate of the film at the interface; this gradient determines the tangential stress at the interface. The gradient depends not only on the kinetic energy of the gas, but also on the roughness parameters of the film, related to the characteristic dimensions of the waves. Furthermore, we cannot neglect the fact that the interfacial friction at the interface is also governed by turbulent pulsations. On the basis of these observations, it could be suggested that the friction at the interface is governed primarily by the Reynolds numbers of the two phases.

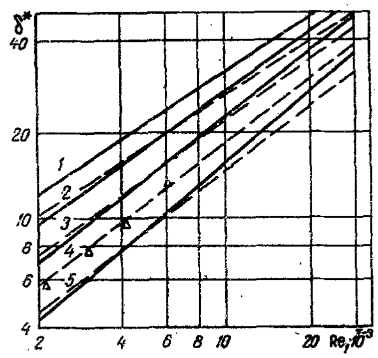

Fig. 3. Average film thickness as a function of the Reynolds number of the film for various tangential stresses at the interface. 1) $\tau^{\delta *}=5$; 2) 10; 3) 30; 4) 50; 5) 100. The points are experimental [8] for $\tau_\delta^*=41$-61.

Figure 2 shows the calculated dimensionless frictional stress at the interface for various values of the Reynolds numbers of the gas and liquid. The dimensionless tangential stress shown in this figure was determined from

$$\tau_\delta^* = \tau_\delta (\mu^2 g^2 \rho)^{-1/3}. \tag{7}$$

The dependences shown in Fig. 2 are described by

$$\tau_\delta^* = 1.35 \cdot 10^{-9} \mathrm{Re}_1^{0.75} \mathrm{Re}_2^{1.75}, \tag{8}$$

from which we see that the surface friction in the two-phase annular flow is largely governed by the kinetic energy of the gas. The magnitude of the surface friction depends on the gas velocity, raised to the same power as in the dependence of the frictional loss determined for the case of the motion of a single gas in a tube; i.e., the frictional stress is proportional to $v_2^{1.75}$.

A comparison with experimental data can tell us how well Eqs. (3) and (4) give the tangential stress at the wall and at the interface. There is essentially no experimental information available from direct measurements of the tangential stresses in turbulent, annular, two-phase flows; the best we can do is make an indirect comparison of these dependences with experiment, for example, on the basis of the thickness of the liquid film.

Figure 3 shows the dimensionless average film thickness as a function of the Reynolds number of the liquid for various values of the frictional stress at the free surface of the film. Since there are no other data available on the average film thickness for conditions corresponding to the present experiments, we used the theoretical results of Dukler [6] for a comparison (the dashed lines in Fig. 3). These results have been confirmed experimentally [7, 8]. As a parameter we choose the dimensionless stress determined from Eq. (7). As we see from this figure, these data agree within 15% over the range of Re_1 studied. Furthermore, our results agree qualitatively with the data of [5], but we cannot make the corresponding comparison because of the difference between the definitions of the Reynolds number of the liquid phase.

The dependence of the film thickness on the Reynolds number of the film and on the surface frictional stress shown in Fig. 3 can be described by

$$\delta^* = 3.45 \tau_\delta^{*-0.78} \mathrm{Re}_1^{0.135 \lg \tau_\delta^* + 0.54}. \tag{9}$$

Equation (9) is a bit awkward to work with, so we can hardly use the values of the friction at the interface as a governing parameter for the film thickness as in [6, 9]. The simplest and most convenient approach in practice is to use the average film thickness as a function of the Reynolds numbers of both phases.

We note in conclusion that these results hold only for the ranges of Re_1 and Re_2 considered here, in which there is a purely film flow, without rupture or entrainment of drops.

NOTATION

x, r, longitudinal and transverse coordinates, respectively; R, tube radius; $\varepsilon = R - r$, variable; P, pressure; v, velocity; ρ, density; g, acceleration due to gravity; δ, film thickness; μ, dynamic viscosity; τ_0, τ_δ, tangential stresses at the wall and at the interface, respectively; i = 1, liquid component; i = 2, gas; $\mathrm{Re}_1 = 4 v_1 \delta \rho_1 \mu_1^{-1}$; $\mathrm{Re}_2 = 2 R v_2 \rho_2 \mu_2^{-1}$; $\delta^* = \delta g^{1/3} \rho_1^{2/3} \mu_1^{-2/3}$.

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